# SOLUZIONI (26-03-2025)

$$\int_0^{+\infty} \boxed{\frac{1}{x}} \cdot \ln\left(1+\frac{1}{2}\sin x\right) dx$$

$y > 0$

$|\ln(1+y)| < |y|$

$>$

$$\int_0^{2\pi} \ln\left(1+\frac{1}{2}\sin x\right) dx =$$

$$= \boxed{\int_0^{\pi} \dots} + \boxed{\int_{\pi}^{2\pi} \ln\left(1+\frac{1}{2}\sin x\right) dx} -$$

$x = t+\pi$

$$= \int_0^{\pi} \ln\left(1+\frac{1}{2}\sin(t+\pi)\right) dt = \int_0^{\pi} \ln\left(1-\frac{1}{2}\sin t\right) dt =$$

$$= \boxed{\int_0^{\pi} \ln\left(1-\frac{1}{2}\sin x\right) dx}$$

$$= \int_0^{\pi} \ln\left(1+\frac{1}{2}\sin x\right) dx + \int_0^{\pi} \ln\left(1-\frac{1}{2}\sin x\right) dx =$$

$$\int_0^{\pi} \ln\left(1+\frac{1}{2}\sin x\right) + \ln\left(1-\frac{1}{2}\sin x\right) dx =$$

$$= \int_0^{\pi} \ln\left(1-\frac{1}{4}\sin^2 x\right) dx = \underline{C} < 0$$

$$\int_{a}^{+\infty} \frac{1}{x} \cdot \ln\left(1+\frac{1}{2}\sin x\right) dx = \boxed{\int_{1}^{+\infty} \frac{1}{x} \ln\left(1+\frac{1}{2}\sin x\right) dx}$$

$$= \int_{1}^{+\infty} \cancel{\frac{1}{x} \ln\left(1+\frac{1}{2}\sin x\right) - \frac{\frac{c}{2\pi}}{x}} + \boxed{\frac{\frac{c}{2\pi}}{x}} \, dx \quad \frac{c}{2\pi}\int_{1}^{+\infty} \frac{1}{x} dx$$

$$\int_{1}^{+\infty} \frac{1}{x} \left( \ln\left(1+\frac{1}{2}\sin x\right) - \frac{c}{2\pi}\right) dx \rightarrow \text{CONVERGE}$$

$$\int_{2\pi}^{4\pi} \ln\left(1+\frac{1}{2}\sin x\right) - \frac{c}{2\pi} \, dx = c - c = 0$$

$$\boxed{\int_{a}^{+\infty} f(x)\, dx}$$

$$\int_{a}^{+\infty} g(x)\, dx \quad \text{Converge}$$

$$\boxed{\int_{a}^{+\infty} f(x) + g(x)\, dx}$$

$$\leq V$$

$$\int_{1}^{+\infty} \left(\frac{1}{x^2} \left|\ln\left(1+\frac{1}{2}\sin x\right)\right|\right) dx$$

$$\leq \frac{k}{x^2}$$

$$\int_1^{+\infty} \boxed{\frac{1}{x^\alpha}}\, dx \quad \begin{cases} \text{CONV.} & \alpha > 1 \\ \text{DIV.} & \alpha \leq 1 \end{cases}$$

$$\int_2^{+\infty} \frac{1}{x^{\alpha}\, \cancel{(\ln x)}^{\beta}}\, dx \quad \begin{cases} \alpha > 1 & \text{CONV.} \\ \alpha = 1 & \begin{cases} \beta > 1 & \text{CONV} \\ \beta \leq 1 & \text{DIV.} \end{cases} \\ \alpha < 1 & \text{DIV.} \end{cases}$$

$\alpha > 1$

$\alpha = 1 + \delta$

$\delta > 0$

$$\int_2^{+\infty} \frac{1}{x^{1+\delta}\, \cancel{(\ln x)}^{\beta}} = \frac{1}{\left(x^{1+\frac{\delta}{2}}\right) \boxed{x^{\frac{\delta}{2}} (\ln x)^{\beta}}} \leq \frac{1}{x^{1+\frac{\delta}{2}}}$$

$\boxed{x^{\frac{\delta}{2}} (\ln x)^{\beta}} \to +\infty$

$\beta < 0 \quad \dfrac{x^{\frac{\delta}{2}}}{(\ln x)^{|\beta|}}$

$$\leq \int \frac{1}{x^{1+\delta}}$$

$\alpha < 1 \qquad \delta > 0$

$\alpha = 1 - \delta$

$$\int_2^{+\infty} \frac{1}{x^{1-\delta} (\ln x)^{\beta}}$$

$\beta \leq 0 \qquad \dfrac{1}{x^{1-\delta} (\ln x)^{\beta}} \geq \boxed{\dfrac{1}{x^{1-\delta}}}$

$$x^{-\frac{\delta}{2}} (\ln x)^{\beta} = \frac{(\ln x)^{\beta}}{x^{\frac{\delta}{2}}} \to 0$$

$\beta > 0 \qquad \dfrac{1}{x^{1-\frac{\delta}{2}} \cdot x^{-\frac{\delta}{2}} \cdot (\ln x)^{\beta}} \geq \boxed{\dfrac{1}{x^{1-\frac{\delta}{2}}}}$

$$\int_{\ldots}^{+\infty} \frac{1}{x\,|\ln|x||^\beta}\,dx \overset{x=e^t}{=} \int_{\ln 2} \qquad \int_0^{\frac{1}{2}} \frac{1}{x^\alpha|\ln|x||^\beta}$$

$$\|$$

$$\lim_{b\to+\infty}\int_2^b \frac{1}{(\ln x)^\beta}\boxed{\frac{1}{x}\,dx} = \lim_{b\to+\infty}\int_{\ln 2}^{\ln b} \frac{1}{y^\beta}\,dy = \int_{\ln 2}^{+\infty}\frac{1}{y^\beta}\,dy$$

$$\leftarrow \qquad y := \ln x$$

$$\int \frac{1}{\boxed{x^\alpha \ln x}}\,dx \qquad \int \frac{1}{x^\alpha}$$

$$\int_1^{+} \frac{1}{f(x)}$$

$$x = r(f(x))$$

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$$\int_{e^e}^{+\infty} \frac{1}{y\,|\ln y|\,|\ln(\ln y)|^\gamma}\,dy = \lim_{b\to+\infty}\int_{e^e}^{b} \frac{1}{\ln y\cdot(\ln(\ln y))^\gamma}\,\frac{1}{y}\,dy =$$

$$= \lim_{b\to+\infty}\int_e^{\ln b} \frac{1}{x\,(\ln(x))^\gamma}\,dx = \int_e^{+\infty}\frac{1}{x\,(\ln x)^\gamma}\,dy$$

$$x := \ln(y)$$

$$\boxed{\int_0^{\frac{1}{2}} \frac{1}{x^\alpha |\ln x|^\beta}\, dx}$$

$$\int_{\frac{1}{2}}^{1} \frac{1}{\cancel{x^\alpha} |\ln x|^\beta}\, dx$$

$$\int_1^{e} \frac{1}{x^\alpha (\ln x)^\beta}\, dx$$

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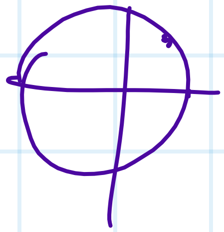
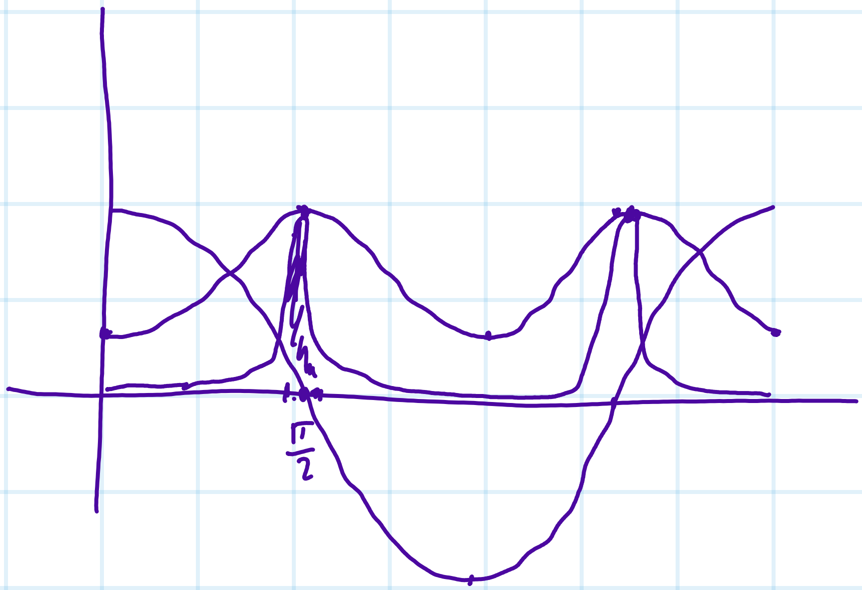
$$\int_0^{+\infty} \left|\sin(\sin x)\right|^x\, dx \qquad \int_0^{+\infty} \left|\cos(\ln x)\right|^x\, dx$$

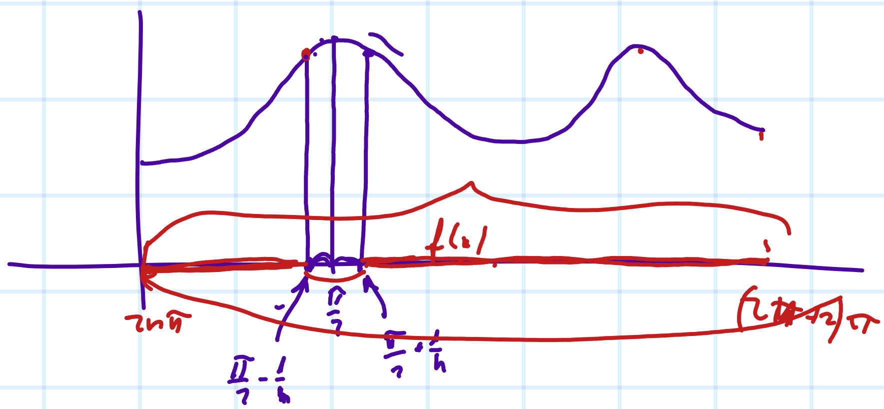
$$\left|\sin(\sin x)\right|^x \leq (\sin 1)^x$$






$$\left|\cos(\cos(x))\right|^n \geq (f(x))^n$$

$$\boxed{\cos\left(\cos\left(\frac{\pi}{2}+\frac{1}{n}\right)\right)}$$




$$F(x) = \int_0^x \quad dx$$

$$\lim_{n\to+\infty} \int_0^{2\pi n} |\cos(\cos x)|^n \, dx \quad = \quad \int_{2k\pi}^{(2k+2)\pi} (\cos(\cos x))^n \, dx$$

$$\lim_{b\to+\infty} F(b)$$

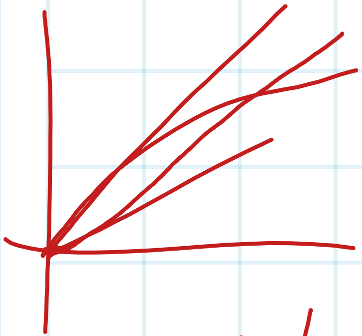
$$\lim_{n\to+\infty} F(n)$$

$$\lim_{n\to+\infty} \sum_{k=0}^{n-1} \boxed{\int_{2k\pi}^{(2k+2)\pi} (\cos(\cos x))^n \, dx} \geq \frac{1}{k}$$

$$\geq \int_{2k\pi}^{(2k+2)\pi} (f(x))^n \, dx \geq \frac{2}{n}\cdot \cos\left(\cos\left(\frac{\pi}{2}+\frac{1}{n}\right)\right)$$

$$[(k+2)\pi]$$

$$(f(x))^{-n} \geq \left|\left| f(x)\right|^{(2k+2)\pi}\right.$$



$$\sum_{k=0}^{n-1} \frac{2}{k}\cdot\left(\cos\left(\cos\left(\frac{\pi}{2}-\frac{1}{k}\right)\right)\right)^{(2k+2)\pi}$$

$$\cos\frac{\pi}{2}\cos\frac{1}{n}+\sin\frac{\pi}{2}\sin\frac{1}{n}$$

$$\cos\left(\sin\frac{1}{n}\right)=1-\frac{\left(\sin\frac{1}{n}\right)^2}{2}\ldots$$

$$\cos\left(\sin\frac{1}{k}\right)\geq 1-\frac{\left(\sin\frac{1}{k}\right)^2}{2}\geq 1-\frac{\frac{1}{2k^2}}{2}=\boxed{1-\frac{1}{4k^2}}$$

$$\geq \sum_{k}\frac{2}{k}\left(1-\frac{1}{4k^2}\right)^{(2k+2)\pi}\geq\frac{1}{k}$$

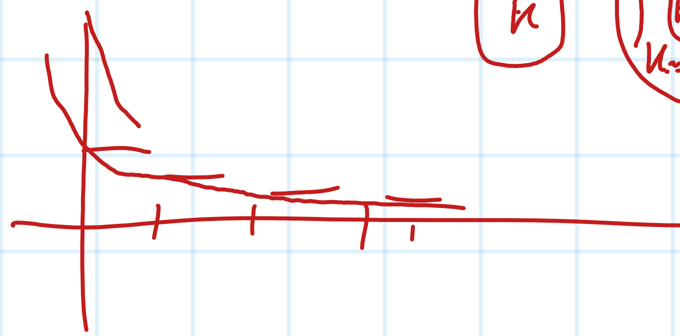
$$\left(\left(1-\frac{1}{4k^2}\right)^{4k^2}\right)^{\frac{(2k+2)\pi}{4k^2}}\to 0$$

$$\frac{1}{e}$$

$$\sum_{k=1}^{n} \frac{1}{k} = \sum_{k=1}^{n} \int_{k-1}^{k} \frac{1}{x+1}\, dx = \int_{-1}^{n} \frac{1}{x+1}\, dx = \text{f.i.o.}$$

$$\frac{1}{k} = \int_{k-1}^{k} \frac{1}{k}\, dx = \int_{k-1}^{k} \frac{1}{\lfloor x \rfloor + 1}\, dx$$

$$= \int_{k-1}^{k} \frac{1}{\lfloor x \rfloor + 1}\, dx$$



$$\int_0^n \frac{1}{x+1}$$

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$$\int_0^{+\infty} \frac{\sin x^2}{x}\, dx \qquad \int_1^{+\infty} \frac{\boxed{\sin(x^2)}}{x}\, dx \quad \text{converge}$$

$$\boxed{F(x) = \int_1^x \sin(t^2)\, dt}$$

$$\lim_{x \to +\infty} F(x) = \text{finite}$$

$$\boxed{\int_1^{+\infty} \sin(t^2)\, dt} \quad \text{converge}$$

$$= \lim_{b \to +\infty} \int_1^b \sin(x^2)\, dx \overset{x = \sqrt{t}}{=} \lim_{b \to +\infty} \int_1^{b^2} \frac{\sin t}{\sqrt{t}}\, dt = \int_1^{+\infty} \frac{\sin t}{\sqrt{t}}\, dt.$$

$$\lim_{b\to+\infty}\int_1^b \frac{\sin x^2}{x}\,dx \overset{x=\sqrt{t}}{=} \lim_{b\to+\infty}\int_1^{b^2} \frac{\sin t}{\sqrt{t}}\cdot\frac{1}{2\sqrt{t}}\,dt = \int_1^{+\infty} \frac{\sin u}{2u}\,du$$

$$\frac{\sin t}{2t}$$

$$\int_1^{+\infty} \boxed{\frac{\sin(x^2+e^{-x})}{x}}\,dx = \int_1^{+\infty} \left(\frac{\sin(x^2+e^{-x})}{x} - \frac{\sin x^2}{x}\right) + \boxed{\frac{\sin x^2}{x}}\,dx$$

$$\int_1^{+\infty} \frac{\sin x^2}{x}$$

$$\int_1^{+\infty} \left|\frac{\sin(x^2+e^{-x}) - \sin x^2}{x}\right| dx$$

$$|(A+B)-A|$$

$$|\sin(A+B) - \sin(A)| \leq |B|$$

$$\int_1^{+\infty} e^{-x}\,dx < +\infty$$

$$\frac{|\sin(x^2+e^{-x}) - \sin(x^2)|}{x} \leq \frac{|e^{-x}|}{x} = \frac{e^{-x}}{x} \leq \boxed{e^{-x}}$$